Introduction to the Program

Solving Math Word Problems: Teaching Students with Learning Disabilities Using Schema-Based Instruction is a teacher-directed program designed to teach critical word problem–solving skills to students with disabilities in the elementary and middle grades. The program is carefully designed to promote conceptual understanding using schema-based instruction (SBI) and provides the necessary scaffolding to support learners who struggle with math word problems. The program features are consistent with national and statewide school reform movements toward challenging academic standards and with the regulations of the No Child Left Behind Act of 2001 (NCLB) and the Individuals with Disabilities Education Improvement Act of 2004 (IDEIA). For example, IDEIA requires access to the general education curriculum for students with learning disabilities. This means ensuring a shift in focus from rote memorization of mathematical procedures to conceptual understanding of mathematical concepts, skills, and relations.

Solving Math Word Problems is organized around two strands: (a) solving addition and subtraction problems and (b) solving multiplication and division problems. The program includes the following features.

- The program focuses on the "big ideas" or salient problem schemata involved in solving addition, subtraction, multiplication, and division word problems. The problem schemata that pertain to a wide range of problems involving all four operations include Change, Group, Compare (additive), Multiplicative Compare, and Vary (Marshall, Pribe, & Smith, 1987). A schema as a knowledge structure serves the function of knowledge organization. According to Marshall (1995), schemata are the basis for understanding and the appropriate mechanism for the problem solver to "capture both the patterns of relationships as well as their linkages to operations" (p. 67). These problem types characterize word problems typically found in elementary and middle grades and are the emphasis of this program (Van de Walle, 1998).
- The program features tasks based on word problems presented in commonly adopted U.S. mathematics textbooks. Word problems are varied and are formatted as text, graphs, tables, and pictographs.
- The program has instruction addressing both conceptual knowledge and procedural knowledge, which are critical to

successful mathematical problem solving (e.g., Hegarty, Mayer, & Monk, 1995). Conceptual knowledge requires problem comprehension and representation, which involve translating the text of the problem into a semantic representation, based on an understanding of the problem type. Whereas procedural knowledge of operations is important, instruction should facilitate "a highly integrated understanding of the operations and the many different but related meanings these operations take on in real contexts" (Van de Walle, 1998, p. 117). The big ideas for developing meanings for the operations should, for example, show that addition and subtraction are connected. In *Solving Math Word Problems,* for example, SBI uses schemata diagrams to represent the information in word problems and to help students figure out what operation is needed to solve the problem (Van de Walle, 1998).

- The program provides appropriate scaffolding of instruction, including the following:

    1. Teacher-mediated instruction is followed by paired-partner learning and independent learning activities.
    2. The first lesson in each unit contains story situations, and subsequent lessons in each unit contain word problems with unknown information.
    3. Initially, diagrams and checklists are provided to support student learning. Later, students construct their own diagrams.

- The program provides adequate practice and a mixed review of problem types.
- Instruction is aligned with national standards in terms of problem solving, communicating, connecting, reasoning, and representing word problems.
- Progress-assessment measures are provided to monitor students' progress in solving word problems.

Program Components

The program is divided into units and lessons. Each unit introduces a problem type—that is, change, group, compare, multiplicative compare, or vary—and the first lesson in each unit contains problem schema instruction for that problem type. Subsequent lessons within each unit focus on problem solution instruction for that problem type. Each lesson begins with a list of materials needed for that lesson (provided as printable forms on the accompanying CD-ROM), a teacher-scripted procedure (contained in this manual), and answers to the worked-out problems (see the Answer Sheets and Reference Guides on the CD-ROM).

During the *problem schema instruction phase* (i.e., the first lesson in each unit), students are provided with *story* situations that contain only known information. They are taught to identify the problem schema (e.g., change, group, compare) and to represent the features of the story situation using schematic diagrams. The aim of this phase is to show students how to understand the underlying structure of the problem type. Students first learn to interpret and elaborate on the main features of the story situation. Next students map the details of the story onto the schema diagram. This step ensures that all irrelevant information in the story is discarded and that problem representation is based on schema elaboration knowledge.

During the *problem solution phase* (i.e., subsequent lessons in each unit), students learn to solve problems with unknowns. A four-step strategy checklist with the acronym FOPS—Find the problem type, Organize the information in the problem using the diagram, Plan to solve the problem, and Solve the problem—is used to anchor students' learning of the word problem–solving strategy to solve word problems (see the Checklists folder on the CD-ROM). Eventually, schematic diagrams are systematically faded at the end of the instructional unit on each problem type.

Following is a list of the printable forms that are provided on the accompanying CD-ROM to support the teaching of the program:

- *Diagrams* for all problem types (use as write-on transparencies; make enlarged copies and either laminate and use as write-on posters or copy on card stock for classroom display)
- *Checklists* for all story and problem types (use as write-on transparencies; make enlarged copies and either laminate and use as write-on posters or copy on card stock for classroom display; pass out individual laminated copies for students)
- *Overhead Modeling* of stories and problems (use as write-on transparencies; pages contain diagrams and space to write out, explain, and model problem-solving processes on the overhead)
- *Reference Guides* for particular lessons (use as transparencies to efficiently illustrate how the problem was solved following a verbal explanation and discussion)
- *Answer Sheets* (make copies and pass out to students to use during paired learning; students can use sheets to correct their errors)
- *Student Pages* (make copies of worksheets that students are required to complete for lessons)
- *Progress Assessment* (print and pass out to students; samples of math word problem–solving forms for monitoring students' progress; results from these assessments can be used to inform instruction and practice)

Using the Program

Although a scripted, detailed teaching procedure is provided in this manual to ensure consistency in implementing the critical content, we recommend that you use the scripts only as a framework for instructional implementation. We do not recommend reading the script verbatim but rather suggest becoming familiar with the script and then using your own explanations and elaborations to implement SBI.

The program can be used whenever students are to solve arithmetic word problems that involve addition, subtraction, multiplication, or division. It can be used in varied settings (general education programs, Title I programs, special education programs) and is designed for flexible use with children of varied needs, primarily those who are at risk for mathematics failure or who may have learning, attention, organizational, and memory difficulties. It can be implemented with individual students or during small- and whole-group instruction. Consider the following conditions, however, when implementing the program:

- *Are students exposed to several problem-solving strategies (e.g., working backwards, using a model, guess and check) at the same time?* If so, the benefits of SBI may be compromised for students with disabilities, who may experience cognitive information overload (see Jitendra, DiPipi, & Grasso, 2001).
- *What is the difficulty level of the word problems when students are first introduced to SBI?* If the problems are too difficult for students, their ability to understand and map the information onto the schematic diagram could be undermined. The goal is for students to learn how to use the strategy; therefore, initial problems should be ones that students are able to read and understand.
- *When cooperative learning groups are employed during SBI, have students received sufficient time to master the new material individually?* This is important to prevent students with disabilities from assuming a passive role in the group (see Jitendra et al., 2001).
- *When implementing SBI in general education classrooms, do some students need more intense and systematic instruction than others?* Teachers should consider the importance of appropriately mediating instruction (e.g., providing extended practice, additional explanations and elaborations) for students with disabilities to be successful problem solvers (see Jitendra et al., 2001; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, in press).

Guide to Paired Learning

- *Assign partners before the assignment.* By assigning partners before the activity, instructional time loss is minimized. Partners should be heterogeneously grouped by achievement. For example, rank-order individuals in the group according to their math performance, and then divide the group in half. Pair the top performer in Group 1 with the top performer in Group 2.

- *Change partners.* By changing partners, students get the opportunity to work with other students in the class.

- *Monitor discussions.* It is important that the students work in an area that you can easily access. By circulating around the classroom and monitoring student work, you will be able to assess student comprehension of the word problem–solving task. This can be beneficial for deciding when to reteach specific information or whether to provide immediate remediation. In addition, if one student in a pair appears to be doing all the work, you can address the situation accordingly.

- Teach students to use a think–plan–share procedure.

  Think: Have each student in the pair independently read the problem and think about the features of the word problem to figure out the problem type.
  Plan: Have each student plan to solve the problem by organizing the information using the given diagram and then solve it.
  Share: Ask students to share their plans and answers with their assigned partners. Students with different answers should discuss how each solved the problem and correct their errors using the Answer Sheet. This may be a good time for one student to role-play the teacher and the other the student.

Program Audience

The addition and subtraction word problem–solving lessons are designed for third graders but can be used with second graders by modifying the difficulty level of the language and computation skills. In addition, the lessons can be used with older children who have experienced consistent difficulties in solving addition and subtraction word problems.

The multiplication and division problems are appropriate for middle school students (e.g., fifth to eighth graders). This program can serve as a supplement to word problem–solving instruction presented in published mathematics textbooks.

Solving Math Word Problems is designed primarily for school practitioners (e.g., special education and general education teachers, school psychologists, supervisors). In addition, the program is useful for teacher educators during preservice training (e.g., for undergraduate- and graduate-level teaching methods courses) or in-service training (e.g., as a desk reference for professionals).

Lesson Time Frames

Each lesson is designed to require about 50 to 60 minutes. Some lessons (e.g., mixed review of one-step problems) are shorter (about 30 minutes). If a lesson or problem is not completed in one class period, it can be completed on the following day.

Judging Program Effectiveness

An effective way to determine whether or not SBI is working is to frequently question and evaluate students' performance at the end of each lesson. This evaluation serves to check students' problem-solving knowledge and determine whether they mastered the strategy steps. When students incorrectly respond to a word problem, review their work to examine possible errors related to strategy use. This information can be used to provide corrective feedback and make modifications (e.g., by modeling more examples or rephrasing the word problem). Examine students' independent worksheets for strategy use (e.g., drawing a diagram, mapping information onto the diagram, planning, writing the number sentence) and provide them with additional instruction as needed before moving to the next problem type. Over time and with frequent practice, students should be able to explain the features of the problem types and verbalize the strategy steps as they solve different problems.

Administer a progress assessment measure (see accompanying CD-ROM) once every 1 to 2 weeks to monitor student performance. In addition, evaluate students' maintenance of strategy use over time and transfer of problem-solving skills to solve novel and complex problems. Finally, assess student satisfaction to determine the benefits of SBI. If SBI is not having the desired effects, ask yourself the following questions:

- Has the student mastered the prerequisite skills (e.g., identifying the different problem schemata) to a criterion level?
- Has sufficient modeling of strategy steps using several examples and explanations been provided?
- Has systematic and varied practice been provided?

What You Need To Know About the Program

The program requires comprehensive, teacher-directed instruction to promote student success. It also depends on teacher orchestration of classroom management skills, delivery of instruction, and fidelity to the program. Although instruction should be explicit, it is equally necessary that you employ frequent student exchanges (opportunities for student responses) to facilitate the identification of critical elements of the problem schema. As such, checking student understanding and providing appropriate feedback on strategy usage on an ongoing basis is essential. For example, when students inaccurately identify the problem type, remind them to check the strategy steps and apply them in the correct sequence.

As with any instructional program, time is necessary for the program to be effective for students with learning problems. The more instruction and practice students receive, the more likely they are to make strong progress. Scheduling sufficient review time is necessary to facilitate acquisition and maintenance of the taught skill.

The program also requires differentiation of instruction based on student ability levels; low-performing students, in particular, may need more instructional support (e.g., explicit instruction, diagrams, checklists) than others to reach their potential. Instruction should be criterion based rather than time based for these students. Ensure that students are proficient in verbalizing the strategy steps and solving problems using schemata diagrams prior to removing the checklists and diagrams. All students need to be exposed to a variety of problems to promote generalization of the problem-solving skill.

Research Evidence To Support the Program

A growing body of literature on mathematical problem solving provides empirical support for SBI (e.g., Jitendra & Xin, 1997; Xin & Jitendra, 1999). SBI is known to benefit elementary, middle, and high school students with learning disabilities and students at risk for math failure (Hutchinson, 1993; Jitendra et al., 1998; Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck, 1999; Zawaiza & Gerber, 1993), as well as students without disabilities (e.g., Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Fuchs et al., 2003a; Fuchs et al., 2003b; Fuchs, Fuchs, Prentice, et al., 2004; Jitendra et al., 2007; Jitendra et al., in press). The following studies represent a decade of research that has examined the effectiveness of SBI for enhancing students' mathematical problem-solving skills: Jitendra et al. (2001); Jitendra, DiPipi, and Perron-Jones (2002); Jitendra et al. (in press); Jitendra et al. (2007); Jitendra et al. (1998); Jitendra and Hoff (1996); Jitendra et al. (1999); and Xin, Jitendra, and Deatline-Buchman (2005).

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